Lesson 3 2-Period Model (2)

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GDP and GNI

- Because investment is introduced into the model, the definition of current account shall change. Although all of wealth in the small open economy corresponds to the external assets previously, the wealth corresponds to the sum of the external assets and the capital.
- Thus, the wealth in the end of period t is $B_{t+1}+K_{t+1}$. In this case, the relationship between a change in the wealth and the saving is:

$$B_{t+1}+K_{t+1}-(B_t+K_t)=Y_t+r_tB_t-C_t-G_t$$

- Plugging this and Eq.(3.12) into Eq.(3.10) yields the definition of the current account when the investment exists in as follows:

$$\begin{aligned} CA_t &= B_{t+1}-B_t \\ &= S_t-I_t \end{aligned} \qquad (3.14)$$

where $S_t \equiv Y_t+rB_t-C_t-G_t$ denotes the national saving. The second line in Eq.(3.14) shows that the difference between the saving and the investment corresponds to the current account.

- The sum of current account and capital account is definitely zero if foreign reserve is negligible. This implies that the surplus in current account definitely corresponds to the deficit in capital account.
- Thus, Eq.(3.14) implies that an increase in the external assets corresponds to the deficit in capital account.

- Now, the concept of current account in the model is consistent with the actual concept because of investment and we mention the relationship between national income account and current account.
- The definition of Gross Domestic Product is given by:

$$Y_t \equiv C_t+I_t+G_t+NX_t \qquad (3.15)$$

Where $NX_t \equiv EX_t-IM_t$ denotes the net export and EX_t and IM_t denote the export and the import, respectively.

- By combining Eqs.(3.14) and (3.15) and the definition of the national saving yields:

$$CA_t = NX_t+rB_t$$

This equality shows that the current account corresponds to the sum of the net export and the interest income.

- GNI is given by Y_t+rB_t Thus, combining this and Eq.(3.15) yields:

$$Y_t+rB_t=C_t+I_t+G_t+CA_t$$

where C_t+I_t+G_t is absorption.

- By comparing this with Eq.(3.15), we can understand that GDP is the sum of absorption and net export while GNI is the sum of absorption and current account.

Investment and Households' Maximization Problem

- Introducing investment into the model changes households' budget constraint. Thus, we reconsider households' maximization problem.
- Paying attention to B_1=0, Eq.(3.14) can be rewritten as:

$$B_2 = Y_1 - C_1 - G_1 - I_1$$

- Similarly, because of B_3=0, Eq.(3.14) also yields:

$$-B_2 = Y_2 + rB_2 - C_2 - G_2 - I_2$$

- By combining these 2 equalities, we have households' intertemporal budget constraint as follows:

$$C_1 + I_1 + \frac{C_2 + I_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}$$

- Households maximize Eq.(3.1) subject to Eq.(3.16). While the budget constraint contains the government expenditure and the investment, still the FONC Eq.(3.3) and Euler equation Eq.(3.4) are applied because households just control the consumption.

Production Possibility Frontier and Equilibrium

- Now, we focus not only indifferent curves and budget constraints but also production possibility frontier under an economy without international indebtedness.
- Production possibility frontier is combinations of feasible and efficient amount of production
- The Production possibility frontier without international indebtedness is given by:

$$C_2 = F\left[K_1 + Y_1 - C_1\right] + K_1 + Y_1 - C_1 \qquad (3.17)$$

- Let assume that this economy choose minimum investment in period 1 and consumes all of accumulated capital, that is, the economy chooses I_1=$-K_1$.
- In this case, the consumption period 1 is $C_1 = K_1 + F(K_1)$, which is extremely high.
- However, because of zero capital accumulation, the consumption in period 2 is C_2=0.

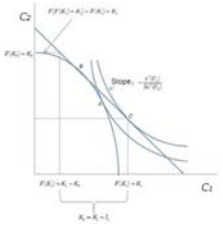
- On contrary, households do not consume the capital in the end of period zero K_1を and all of the output in period 1 assign to investment.
- In this case, the consumption and the investment in period 1 are C_1=0 and I_1=$F(K_1)$, respectively.
- In the period 2, those are $C_2 = F\left[K_1 + F(K_1)\right] + K_1 + F(K_1)$ and K_2=K_1+$F(K_1)$, respectively.
- Although the consumption in period 1 is zero, it in the period 2 is extremely high, as shown in the intercept, in this case.

- The slop of the PPF is derived from Eq.(3.17) and is given by:

$$\frac{\partial C_2}{\partial C_1} = -\left[1 + F'(K_2)\right]$$

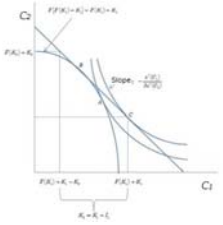
- This implies that the PPF is strictly concave because of diminishing marginal productivity.

Fig. 3-2: the FFP and the Consumtion determined



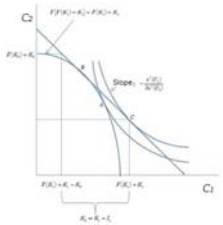
- In point A, the slop of both the indifferent curve and the FFP is $-(1+r^A)$ where r^A はdenotes the real interest rate in a closed economy.
- On one hand, the slope of the budget constraint is given by $-(1+r)$. This implies $r^A > r$.
- In this economy where is international indebtedness is given by r which is the world interes rate.

Fig. 3-2: the FFP and the Consumtion determined



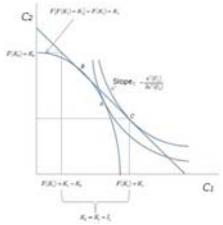
- The higher return rate attracts foreign investors and they start to invest in the home. The production increase and the output goes to point B where the budget constraint crosses with the FFP.
- Similar to Fig. 3-1 analyzing consumption under the model without investment, the consumption is point C where the indifferent curce crosses with the budget constraint

Fig. 3-2: the FFP and the Consumtion determined



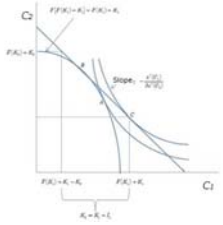
- Fig: 3-2 shows the relationship between the consumption in 2 periods and the current account if there is the investment instead of Fig. 3.1.
- The FFP shows the combination of the highest consumption when there is not international indebtedness.

Fig. 3-2: the FFP and the Consumtion determined



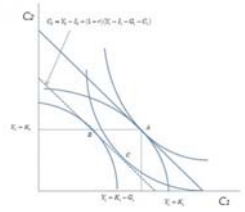
- Thus, the earning rate on domestic investment exceeds the world interest rate in point A.

Fig. 3-2: the FFP and the Consumtion determined



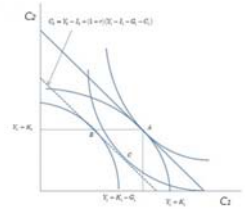
- Fig. 3-2 shows that output is not point A but point B.
- That is, the investment from the foreign enhances households' utility.

Fig: 3-3: The Government Expenditure and the Current Account



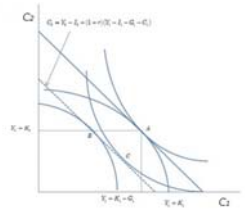
- Now, we introduce not only the investment but also the government expenditure.
- The budget constraint Eq.(3.16) and the PPF Eq.(3.17) explain how a change in the government expenditure affects the consumption.

Fig: 3-3: The Government Expenditure and the Current Account



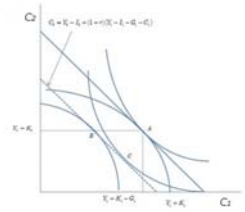
- An increase in the government expenditure in period 1 shifts Eqs.(3.16) and (3.17) left side by the same increase.
- An increase in the government expenditure in period 2 shifts Eqs.(3.16) and (3.17) downward by the same increase.

Fig: 3-3: The Government Expenditure and the Current Account



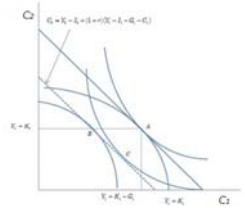
- Point A in Fig. 3-3 shows the consumption when both the government expenditure and the current account are zero.
- This can be understood easily because point A is an intersection of the indifferent curve and the PPF.

Fig: 3-3: The Government Expenditure and the Current Account



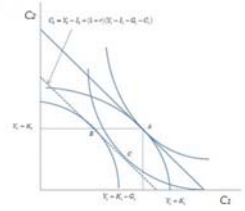
- Let assume $G_1 > T_1$ and $G_2 = 0$. In this case, both the budget constraint and the PPF shift left side by G_1 and the production is point B.
- Because of international indebtedness, the consumption is point C where both shifted budget constraint and the PPF cross.

Fig: 3-3: The Government Expenditure and the Current Account



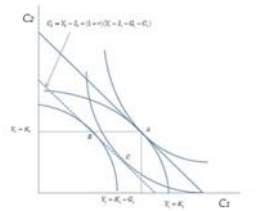
- Thus, An increase in the government expenditure in period 1 induces the current account deficit in period 1.

Fig: 3-3: The Government Expenditure and the Current Account



- An increase in the government expenditure in period 2 shifts both the budget constraint and the FFP downward and induces the current account surplus in period 1.
- While an increase in the government expenditure decreases the disposable income, households make the consumption constant over time.

Fig: 3-3: The Government Expenditure and the Current Account



- Thus, the current account deficits or surplus occur through international indebtedness.

Fig. 3-1: Fiscal Balance and Current Account

		Fiscal Balance	
		Period 1	Period 2
Current Account	Period 1	Deficit	Surplus
	Period 2	Surplus	Deficit

- Fig. 3-1 shows the relationship between the government expenditure and the current account.

Interest Rate and Intertemporal Consumption

- Eq.(3.3) can be rewritten as:

$$\beta\left(\frac{u'(C_2)}{u'(C_1)}\right)=\frac{1}{1+r} \qquad (3.4)$$

- The LHS in Eq.(3.4) shows the marginal rate of substitution between the consumption in periods 1 and 2.
- The RHS in period 2 shows the price of the consumption in period 2 in terms of the consumption in period 1.

- Eq.(3.3) shows that households' optimal consumption schedule depends on both the subjective discount factor and the marginal rate of substitution on intertemporal consumption.
- For example, when the subjective discount factor is smaller than the marginal rate of consumption, that is, $\beta<1/(1+r)$, $u'(C_1)<u'(C_2)$ is applied.

- Now, $u''<0$ which implies that the marginal utility of consumption is diminishing.
- When the marginal utility is smaller, the consumption is enough.
- Thus, $u'(C_1)<u'(C_2)$ implies that C1>C2, that is, the consumption in period 1 exceeds it in period 2.

- When $\beta<1/(1+r)$ which is corresponding to $r<\delta$ that is, the interest rate is below the rate of time preference, the consumption in period 1 is hiked because households cannot obtain enough reward as refraining consumption.
- When the subjective discount factor is larger than the marginal rate of substitution, $C_1<C_2$, that is, the consumption in period 2 exceeds it in period 1 applied by the similar reason.

- When $\beta = 1/(1+r)$ is applied, that is, the subjective discount factor corresponds to the marginal rate of substitution, Eq.(3.3) boils down to $u'(C_1) = u'(C_2)$.
- This implies that $C_1 = C_2$ and the consumption in period 1 equals to it in period 2.
- That is, the consumption is constant over time.

- If the subjective discount factor does not correspond to the inverse of the (gross) interest rate 1/(1+*r*), the consumption in period 1 does not corresponds to it in period 2 because the marginal utility of consumption in period 1 is different from it in period 2.
- This implies that controlling consumption is possible through controlling the interest rate.

- When the interest rate in period 1 is hiked, for instance, the marginal rate of substitution falls.
- In that case, the consumption period 2 is come under pressure to decrease.

$$\downarrow \beta\left(\frac{u'(C_2)}{u'(C_1)}\right) = \frac{1}{1+r_1\uparrow} \downarrow$$

- A decrease in the marginal utility of consumption increases the consumption.
- In this case, the consumption period 2 is about to increase relatively.
- This fact implies that monetary policy can affect consumption.
- Of course, the (real) interest rate in the home corresponds to it in the foreign. Thus, there is not such a change.
- However, by introducing some modification, the model can analyze how monetary policy affects consumption (or investment).

Problems

1. By introducing the government expenditure, households' budget constraint is given by Eq.(3.9) instead of Eq.(3.2). Derive Eq.(3.9) by using Eq.(3.2).
2. The production function is given by $Y_t = A_t F(K_t)$ instead of Eq. (3.11) where A_t is productivity. Now, let imagine that the productivity in period 1 increases. Illustrate how a change in the PPF affects the current account in periods 1 and 2. Note that the government expenditure can be ignore.

3. Fig. 3-2 shows that international indebtedness enhances consumption rather than it in closed economy. However, as shown in Fig. 3-1, this enhance accompanies with current account deficit in period 1. Recently, current account in balance in countries all over the world, namely, global imbalance is concerned. In the text, we point out that this concern is not pertinent. Why is this not? Is this concern is definitely irrelevant?